Tensile Strength

The tensile strength of paper sheets is especially complex as many variables play a role in controlling the magnitude of this property. Tensile strength is dependent on both the fiber strength properties and the bonding that occurs between fibers. The tensile strength theory that has attracted the most attention has been that of Page. The "Page" equation was reported in a publication in 1969 and remains a cornerstone in paper physics. The equation represents a comprehensive account of the variables encountered in attempting to predict tensile strength from the properties of the fiber and for bonds between fibers.

$$\left[\frac{1}{T}\right] = \left[\frac{9}{8Z}\right] + \left[\frac{(12g \cdot C)}{(P \cdot l \cdot b \cdot RBA)}\right]$$

The Page Equation

$$RBA = \frac{(S_0 - S)}{S_0}$$

l	= fiber length (length)
b	= fiber-fiber bond strength (N/m^2)
RBA	= relative bonded area (unit less)
g	= gravitational constant -(length/second^2 = 9.8 m/s^2)
T	= tensile breaking length (length)
Z	= zero span tensile (length)
C	= fiber coarseness (weight/length)
P	= fiber perimeter (length)

The relative bonded area (RBA) in Page's equation is a measure of the contact area between fibers in the sheet. This is measured by light scattering co-efficient or through nitrogen absorption measurements. Increases in bonded area can be achieved by increasing wet-pressing pressure. With subsequent testing of a strength property (such as tensile strength) and scattering coefficient, the sheet strength can be extrapolated to zero sheet strength. The result of this extrapolation is an estimate of the scattering coefficient of unbonded fibers (S_o) that can be used to calculate the relative bonded area. Equation 1 shows the relationship between relative bonded area and light scattering co-efficient.

$$RBA = \frac{(S_0 - S)}{S_0}$$

Equation 1: Page’s equation for computing relative bonded area

S_o = scattering co-efficient of the unbonded sheet (m^2/kg)
S = scattering co-efficient for a paper sheet (m^2/kg)
In the Page equation, most of the variables are measurable except for b, the fiber to fiber bond strength or “shear strength” of the fiber bonds. Once all of the measurable variables are obtained, the Page parameter ([1/T – 9/(8Z)]-1) can be plotted against the light scattering coefficient (S) (Equation 5). This plot can be used to obtain the bond strength (b) and the scattering coefficient of the unbound fibers (S_o) from the slope and intercept respectively.

$$\left[\left(\frac{1}{T}\right) - \left(\frac{9}{8Z}\right)\right]^{-1} = b \cdot \left[\frac{1}{\gamma} - \frac{S}{(\gamma \cdot S_0)}\right]$$

$$\gamma = \left(\frac{12g \cdot C}{P \cdot l}\right)$$

Equation 5: Parameters to plot for obtaining bond strength using the Page equation

Note: The Page equation is only valid for sheets made with good formation, free from kinks or curls. This is because sheets with poor formation fail earlier due to uneven concentrations of stress in areas of low basis weight. Kinks and curls cause changes in the fiber length variable in the equation